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DYNAMIC FRACTURE AND SCALE EFFECTS (SURVEY)

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1. Introduction. The most important result of linear fracture mechanics (LFM) — which is based on Griffith's notion that a crack enters an unstable state — was the recognition of the fact that fracture (the separation of a whole into parts) is in fact the act of work being done. This work is done by elastic strain energy. The acceptance of this point of view led to a critical reexamination of fracture criteria and the development of new methods of testing materials for strength. Traditional strength criteria — the critical values of stress σ_B and strain ε_f and their combination — turned out to be inadequate. The role of material characteristics such as tensile strength σ_B was limited to the comparison of materials in standard tests. Meanwhile, technical cohesive strength turned out to be 2-4 orders of magnitude lower than the actual strength of materials [1]. It became necessary to introduce new fracture criteria, including the fracture work completed on a unit surface G_{1c}. This quantity is usually determined as part of the combination $G_{1c}E \sim K_{1c}^2$ (where E is the elastic modulus and K_{1c} is the critical value of the stress-intensity factor).

The introduction of new fracture criteria required the development of new methods of determining them. The traditional strength criteria were easily found by constructing the stress-strain curve of the material, since these criteria were special points of the curve (Fig. 1). This was point σ_B on line 1 for ductile materials and point σ_f on line 2 for brittle materials. Line 3 is an approximation of line 2 used in the present study, while line 4 is the same as 3 except for a higher strain rate. A series of tests with special specimens of different sizes usually has to be performed in order to obtain the quantities 2γ and G_{1c} or K_{1c} and K_c, etc. in linear fracture mechanics.

However, the major successes of LFM and its modifications notwithstanding, this approach still has certain deficiencies in regard to the practical application of the results of investigations to specific structures. As evidence, we can cite the periodic catastrophic failures of large structures designed in accordance with existing strength standards. The difficulties of using LFM increase sharply when attempts are made to describe high-rate dynamic failure — in which case diagnosing the growth and development of structural defects is almost impossible. These considerations give a reason to look for new approaches to solving fracture problems that differ from the local nature of LFM investigations and studies performed by similar methods. Just as knowing the laws governing the growth of a single tree does not guarantee that the processes involved in the growth of a forest will be understood, so it is that restricting the study of the loading of metals to LFM and its modifications may blind investigators to the principles underlying the failure of the load-bearing members of a structure or the structure as a whole.

2. Integral Approach to the Problem of Fracture. Experiments involving the fracture of geometrically similar (GS) vessels and shells subjected to high-rate loading have confirmed the existence of significant energy-related scale effects (ERSE). The existence of these effects follows from an examination of the elastic energy (EE) balance and the fracture work for the entire object being examined and must be accounted for in order to be able to predict fracture. We will henceforth refer to this approach to the problem as the integral approach (IA), in contrast to the local approach used in the LFM or its modifications. In LFM, the energy balance is examined in the neighborhood of the crack tip.

The IA was used in [2] for a cube of a material with the edge L. The cube was subjected to tension on two opposing faces with the force σL^2. Without loss of generality, it was decided that a bilinear law governed the deformation of the material:

$$\sigma = \varepsilon E \quad \text{at} \quad \sigma \leqslant \sigma_0 \text{ and } \sigma = \sigma_0 + K(\varepsilon - \sigma_0/E) \quad \text{at} \quad \sigma > \sigma_0. \tag{2.1}$$

Here, K and σ_0 is the strain-hardening modulus and elastic limit of the material. Of course, the value of σ_0 for actual materials depends on both the strain rate $\dot{\varepsilon}$ and the change in the temperature T of the material, and the latter may in turn change due

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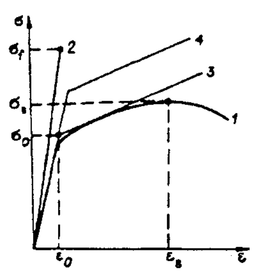

Fig. 1

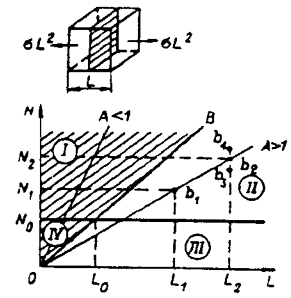

Fig. 2

to plastic flow. However, if the change in scale is not too great and if fracture is examined in the elastic region or the beginning of the elastoplastic region, these effects are small and can be accounted for as corrections. The IA was subsequently generalized to other simple objects.

The elastic region of deformation is the region of the greatest practical interest. Here, fracture occurs in a brittle manner over short intervals of time, and the work done by external forces can be ignored. The same conditions prevail in the case of dynamic failure in the plastic region. However, description of fracture in this case requires consideration not only of the expenditures of elastic energy on plastic flow and the increase in T, but also any possible change in G_{1c} and the elastic constants of the material with an increase in fracture strain ε_f.

For a cube of material, the ratio of elastic energy $L^3\sigma^2/2E$ to the work of rupture $L^2\lambda$ has the form

$$A = \sigma^2 L/(2\lambda E) \tag{2.2}$$

(where λ is the unit work of rupture of the material, the analog of 2γ and G_{1c} in LFM). When $A \geq 1$, Eq. (2.2) is a necessary condition of fracture. When $A < 1$, it is a necessary condition for nonfracture. In the plane N — L (where $N = 2\lambda E/\sigma^2$), the rays emanating from point O (Fig. 2) correspond to the states of the material with a constant value of A. In the region $A < 1$ (NOB), they correspond to states with the same residual strength, while at $A \geq 1$ (BOL) they correspond to states with the same probability of fracture. The value $\sigma = \sigma_0$ corresponds to the line $N = N_0$, which with the ray $A = 1$ divides the plane into four characteristic regions. The intersection of N_0 and $A = 1$ corresponds to $L = L_0$ — the minimum value of L at which the cube can undergo brittle fracture (fracture in the elastic region). This is the brittleness threshold of the material

$$L_0 = 2\lambda E/\sigma_0^2. \tag{2.3}$$

Thus, the transition to the brittle state with an increase in the size of the object is a physical property of the material. The value of L_0 depends on temperature T and loading rate $\dot{\sigma}$ through σ_0, as well as on the stress state. This can be shown by replacing tension by compression. In this case, σ_0 must be replaced by $\nu\sigma_0$, and the value of L_0 increases by the factor ν^{-2}. Thus, the stress at which brittle fracture occurs in compression is 10-20 times greater than the corresponding stress in tension. The value of L_0 in static tension at $T \sim 300$ K for copper, stainless steel, mild steel, quenched steel, organic glass, and silicate glass is 8.3, 1.5, 0.6, 0.05, 0.006, and 10^{-6} m, respectively. The notation L_0 conforms to the formula for Irwin's plastic correction [3].

Of the four regions in Fig. 2, brittle fracture is possible only in region II. While states b_1 and b_2 of two cubes with the edges L_1 and L_2 lie on a ray associated with the same probability of fracture, ERSE will be seen in the fracture of the cubes in accordance with Eq. (2.2):

$$\sigma_1/\sigma_2 = (L_2/L_1)^{1/2} \quad \text{or} \quad \sigma \sim L^{-1/2}. \tag{2.4}$$

In the general case, if the states of cubes undergoing fracture lie on rays associated with different probabilities of failure, the manifestion of ERSE may be somewhat weaker (points b_1 and b_3) or stronger (points b_2, b_4) than would be expected from (2.4).

Can LFM and its modifications predict the occurrence of ERSE? Not in explicit form. For example, the following was written in [4] for a plate of width L with a central slit $2l$

$$K_c = \sigma\sqrt{L\,\mathrm{tg}(\pi l/L)}. \tag{2.5}$$

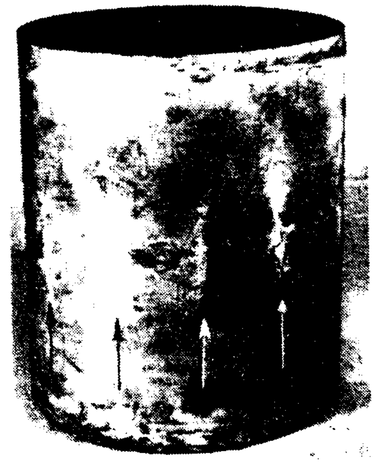

Fig. 3

Following LFM and regarding the slit as a random quantity independent of L, we obtain the below expression after we expand (2 4) into a series

$$K_c = \sigma\sqrt{l\pi}\,[1 + (\pi l/L)^2/3 + \ldots].$$

Thus, the effect of L is expressed by a correction of second-order smallness. The same result follows from an examination of other types of cracks (slits). The imposition of an additional restriction within the framework of LFM (l/L = const) reduces Eq. (2.5) to (2.4), i.e. to ERSE, but at the same time it also reduces the overall static examination of the problem to a special low-probability event. On the other hand, the above limitation converts the defect into an intrinsic feature of the object and makes the study an exercise in the use of the IA [5].

Let us cite one more example that leads us to ERSE, when the Γ—integral in nonlinear fracture mechanics is employed. A criterion for the safety (nonfracture) of pipelines was obtained in [6]. An analysis of this criterion shows that energy-related scale effects are obtained if the nonessential terms of the solution are discarded. This solution follows as an elementary result from the IA [7]:

$$\sigma_\theta < \{E\lambda/[\pi R(1 - \nu^2)]\}^{1/2}$$

(σ_θ and ν is the hoop stress and the Poisson's ratio). The examples considered above that lead to ERSE provide grounds for hoping that condition (2.4) will be an intrinsic part of all solutions of similar problems of LFM and its modifications.

Thus, the scale effect first observed experimentally by Galileo in the fracture of geometrically similar objects and considered by him to be a characteristic property of a material is clearly shown to be a manifestation of ERSE when an energy-based fracture criterion is used.

The transition to the region of plastic deformation which occurs as ε increases is accompanied by a sharp increase in the work done on plastic flow. The latter in turn leads to the growth of structural defects inside the material. The quantities λ, ν, σ_0, and E undergo substantial changes at this stage, and the fraction of elastic energy decreases abruptly relative to the work done on plastic flow. Thus, the expenditure of EE on the joining of scattered defects into a main crack turns out to be relatively small. This situation is equivalent to the absence of ERSE, as is confirmed by the failure of materials far from the yield point in static tests.

3. Dynamic Fracture. When requirements regarding the similarity of the loading equipment are met, geometrically similar objects can readily be shock-loaded by using explosive charges [1]. For GS shells subjected to central or axisymmetric loading, the effect of the explosive can be calculated using a short impact approximation. In this case, the values of σ and ε in the elastic strain region will be proportional to ξ at similar moments of time, where ξ is the ratio of the weight of the explosive charge to the weight of the structure (or a certain part of the structure). Thus, ξ can be regarded as the analog of pressure in the static loading. The rate of increase in σ decreases after the transition to the plastic region, since E >> K. The relation $\sigma(\varepsilon)$ will be weak, and it is best if the relation $\varepsilon(\xi)$ is used instead.

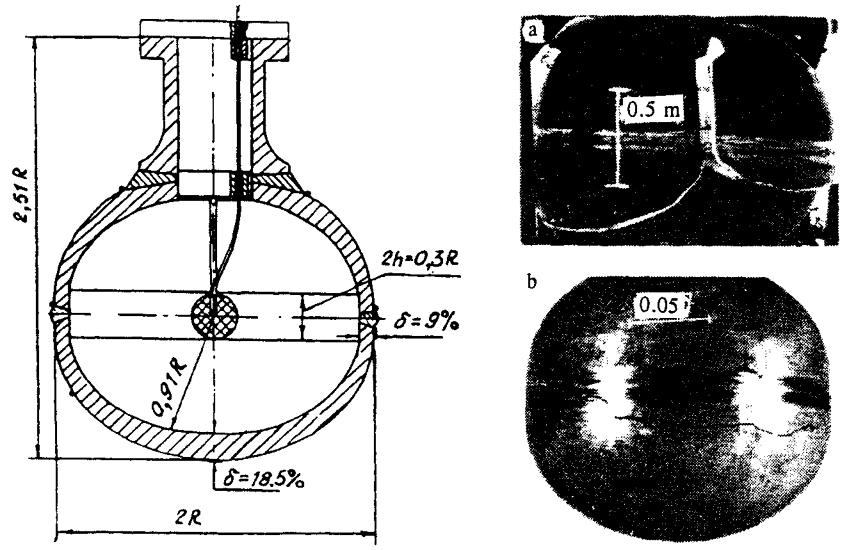

Fig. 4

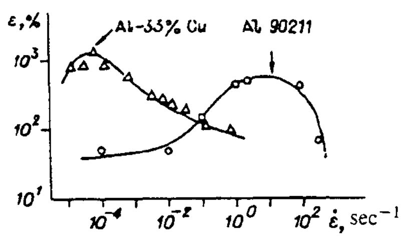

Fig. 5

Wave processes begin to play a more prominent role in dynamic loading as loading rate $\dot{\sigma}$ is increased. The same applies to a lesser extent to the relationship between a given object and the external medium. In fact, sometimes the object itself, rather than being an integrated whole, is actually a system of independent autonomous regions in which fracture processes are localized and independent. In a number of cases, dynamic loading makes it possible to more clearly discern the physical laws that result in fracture. Studies of dynamic loading show that satisfaction of the necessary fracture condition also involves the satisfaction of a sufficient condition. The latter exists due to the intensive growth of defects inside the material as a result of the repeated reflection and interaction of compression and rarefaction waves. This is particularly important if the object is deformed in the plastic region. Of course, the foregoing remarks do not mean that defects and stress concentrators initially present in the material do not affect the load-carrying capacity of the object. As in static loading, they will have the effect of decreasing load-carrying capacity and will cause failure to take place at lower values of A (in the limit $A = 1$). Conversely, an object free of such defects will also be functional at $A >> 1$. However, its failure will be more violent and catastrophic, with larger fragments being produced. Let us consider several examples.

A. In the case of the internal shock loading of a thin cylindrical or spherical shell at $\sigma < \sigma_0$, the radial vibrations of the shell will be unstable and become eventually become flexural. The elastokinetic energy of the shell is redistributed and its density turns out to be dependent on the angle: the maximum value occurs at antinodes, the minimum at nodes. Thus, six different waves are generated in a cylindrical glass-plastic shell with a relative thickness $\delta \sim 5\%$. The shell fractures simultaneously along 12 meridional planes corresponding to the antinodes of bending waves. The density of elastic energy at these antinodes is more than six times greater than the initial value associated with the radial vibrations (Fig. 3). The number of bending waves decreases with an increase in δ, and their reaction to shock loads becomes more difficult to predict with a transition to thick-walled shells of more complex form [8].

B. Of particular interest is high-rate fracture occurring with unidimensional deformation — cleavage. Such fractures are initiated simultaneously at an enormous number of sites and proceed almost synchronously over a certain plane. In the limiting case (in the interaction of rarefaction waves from shock loading), the fracture surface is almost mirror-smooth and the number of sites at which fracture originates is estimated to be approximately 10^8 per square centimeter [9]. Since the store of elastic energy in such fracture is determined by the length of the loading pulse ($\sim L^1$) and since the rupture work is independent of the linear dimension ($\sim L^0$), ERSE should be manifest in cleavage [10, 11]. The fact that the deformation process is unidimensional and the high rate at which loading takes place sharply reduce the value of L_0, so that the failure even of materials that are typically ductile occurs in a brittle manner [9]. Energy-related scale effects are also observed, but are somewhat less pronounced than indicated by (2.4) due to the existence of the relation $\lambda(L)$. With a decrease in L to the interatomic spacing, the value of λ should decrease to double the value of surface tension [1]. Thus, even such a specific type of fracture as cleavage is logically incorporated into the energy approach to fracture.

Of particular interest among the other methods of describing cleavage is the attempt to characterize it from the viewpoint of the kinetic concept of strength. Here, cleavage fracture is represented as an extension of the rupture-strength curve of the material during periods of loading $\tau < 10^{-4}$ sec. However, such an description is unacceptable for several reasons: 1) the transition from the range of times-to-rupture $\tau > 10^{-4}$ sec (static part) to the region $\tau < 10^{-5}$ sec is accompanied by an increase in fracture activation energy U by a factor of 3-6 [10], but such a jump in U seems questionable; 2) part of the curve for $\tau > 10^{-4}$ sec is described by the formula

$$\tau = \tau_0 \exp\{(U - \gamma_0\sigma)/kT\}.$$

It follows from the structure of this formula and the dimensions its constants U and γ_0 that it describes only the prepartory stage of fracture (the accumulation of damages in the material), and although the time interval corresponding to the second (main) stage of fracture is short, it is the branch of the curve for $\tau < 10^{-5}$ sec that corresponds to this stage, i.e. the stage in which the crack advances over a certain plane; thus, it is hardly proper to combine these two branches into a single curve; 3) the part of the curve for $\tau < 10^{-5}$ sec was obtained for conditions under which energy-related scale effects might be manifest. The relation $\tau(\sigma)$ for $\tau > 10^{-4}$ sec was determined on bars of uniform cross section in which ERSE were absent, which also serves to make combining the branches improper. Thus, the representation of $\tau(\sigma)$ as a single curve for the interval of τ from 10^4 to 10^{-8} — as was done in [12] — is incorrect. This subject was taken up later in [13].

C. Let us now discuss the fractures of GS objects in the transitional elastoplastic region. Figure 4 depicts the structure of a vessel that was examined, the scheme employed for its blast loading, and photographs of two fractured GS vessels of different sizes (R = 1 and 0.1 m (a, b)) [14]. The completed study showed that it is impossible to have certain important information beforehand if the possibility of ERSE is not taken into account for vessels of these radii. Specifically, we cannot know the load-carrying capacity of the vessel, its weakest point, or the character of failure (brittle and catastrophic or ductile and avalanche). For example, a decrease in R by a factor of 10 (from 1 to 0.1 m) leads to an increase in ξ by a factor of 5-8, a shift in the "weak" point (the site where the crack starts) from the edge of the neck to the thinner equatorial region of the vessel wall), and a change in the nature of the fracture from brittle to ductile. Here, the value of σ increases from 0.18 to 0.54 GPa and ε_f increases from 0.11 to 0.62%.

A strong manifestation of ERSE has also been observed in the explosive failure of other GS objects. For example, the authors of [15] described the failure of thick-walled vessels filled with air (P = 1 atm) with a decrease in radius by a factor of 15. The value of ξ increased by a factor of 15.7 in this case. Energy-related scale effects were observed in [16] in the rapid collapse of tubes.

D. Use of the integral approach has made it possible to approximately calculate the fracture of a thin ring dispersing in the radial direction in a thick plate at the velocity v = const [17]. In accordance with the IA, no restrictions were placed on the mechanism by which fracture could have occurred. It was assumed that the fracture process began simultaneously in several sections with the beginning of motion of the ring and that fracture occurred as a result of elastic energy being removed by unloading waves diverging from these sections. When the sections in which fracture developed were close enough together for the unloading waves to be superimposed, fracture in these sections was interrupted. The amount of strain occurring prior to failure was determined by the period of time over which the elastic energy necessary for failure was removed. The fracture process could have proceeded independently in several sections. The equation of motion of the ring prior to failure has the form

$$\mu^2\dot{e}^2\varepsilon(\varepsilon + 2)/2 + \dot{e}(2\mu\varepsilon - \alpha) + \ln(\varepsilon + 1) = 0,$$

where $\mu = \mu/\sigma_0$; $\alpha = 4E\lambda/(3c\sigma_0^2)$; $\dot{e} = v/R$; η is the viscosity of the material. The solution found for a tough material predicts a maximum in $\varepsilon(\dot{e})$, this maximum being realized when the strength and viscous forces are the same. Such a maximum was observed in [17-20] for several materials. Maxima of $\varepsilon(\dot{e})$, shifted to a region of lower $\dot{e}$, are also seen in connection with

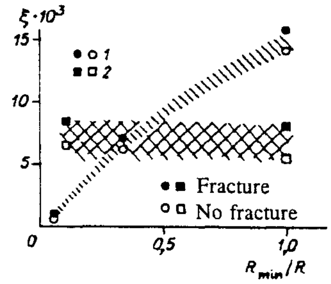

Fig. 6

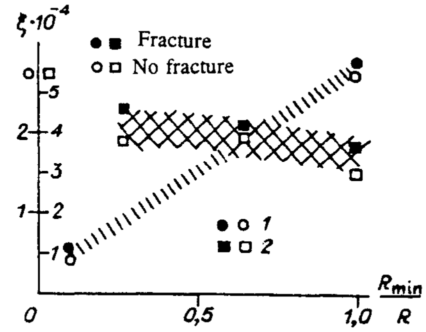

Fig. 7

superplasticity. This follows from the data shown in Fig. 5, which is from [21]. A similar approach was used in [19] to find the solution for spherical shells:

$$\mu^2\dot{e}^2[1 - (1 + \varepsilon)^3]/3 + \dot{e}[\alpha - \varepsilon\mu(\varepsilon + 2)] - \varepsilon = 0.$$

The physical nature of the maximum owes to a sharp increase in elastic strain energy with the inclusion of viscous forces. There is thus a decrease in the size of the region from which the elastic energy needed for fracture is taken, along with a corresponding decrease in time to rupture and the displacement of the shell prior to failure. Later refinement of this approach in [22] made it possible to examine a more general equation for strain and to describe the fragmentation of shells and the breakup of jets from shaped charges. Several researchers [23-26] have stated that the work of rupture is the direct result of the energy associated with inertial forces. An analysis of this subject performed on the basis of theoretical representations and available empirical data indicates that the description of the phenomenon [27, 28] as being related directly to elastic energy should be given priority. As was shown in [11], a similar situation prevails in the description of failures by spallation.

4. Ways to Avoid ERSE. Oriented Fiber Composites. Since new products and structures are often designed on the basis of trial and error, experimentation has shown the way to several approaches that can be taken to avoiding energy-related scale effects. Other methods follow from the integral approach. We will discuss their usefulness below.

A. Let us examine the design of a steel tie to withstand a force F. With regard to the cross section of the tie S, the IA imposes another restriction in addition to the traditional requirement $S \geq F/\sigma_0$. To exclude the possibility of brittle fracture, the characteristic size of a tie of cylindrical cross section $S = \pi D^2/4$ should not exceed the brittleness threshold $L_0 = 2\lambda E/\sigma_0$. Assuming that the diameter $D = \varphi L_0$, we obtain the condition for nonfracture in the elastic strain region:

$$S \leqslant \pi(\varphi E\lambda)^2/\sigma_0^4. \tag{4.1}$$

Here, φ is the proportionality factor with the transition of L_0 from a cube to a long cylinder.

Condition (11) is equivalent to the idea that strong new steels can be used to lighten the weight of ties only when the value of S is small. In order to satisfy the specified value of F, the tie should be replaced by a set of parallel ties of small cross section satisfying (4.1). However, this is what is commonly referred to as a cable.

B. Another way to avoid brittle fracture is to use multilayered vessels or coiled tubes. Such structures are more resistant to brittle cracks than are solid materials. For example, sections of coiled pipes are used in gas lines to stop rapidly growing cracks. Their high crack resistance has also been demonstrated under conditions of intensive dynamic loading [29]. Nevertheless, direct static [30] and dynamic [31] tests of GS objects made of laminated materials have shown that ERSE do occur. However, these effects are weaker than would be expected for structures made of nonlayered materials. For example, a fourfold increase in the size of solid steel pipes reduces ε by a factor of 2-2.5 [32], while for coiled pipes a tenfold increase in size (and a corresponding tenfold increase in the number of layers) reduces ε by a factor of just 1.3 [32]. The reason for the (albeit weaker) ERSE in the latter case is probably acoustic coupling of the layers. In the case of a cable, there is in fact no ERSE.

Energy-related scale effects should be manifest to their full extent if, when a GS change is made in the size of a coiled pipe or multilayered shell, the number of layers is left unchanged and, accordingly, their thickness is changed in proportion to the change in scale.

C. Let us return to the practically important question of the structure of large objects – pressure vessels, heavily loaded shells, etc. Discussed above were results of experiments with GS objects of solid materials that illustrate the occurrence of ERSE. In [33], the IA was used to examine the possibility of brittle subcritical cracking of thin-walled spherical (4.2) and cylindrical (4.3) shells under static (2n = 1) and dynamic loads:

$$\sigma < \{(2n)^{1/2}E\lambda / [2R(1 - \nu)]\}^{1/2} \quad (n = 2{,}12(R/h)^{1/2} - 0{,}5); \tag{4.2}$$

$$\sigma < \{2nE\lambda / [\pi R(1 - \nu^2)]\}^{1/2} \tag{4.3}$$

(n is found from the equation $n^2(n^2 - 1)^2 = 3(n^2 + 1)(R/h + 0.5)^2$, where h is the thickness of the shell). As might be expected, the final formulas can be written using (2.4) or

$$\sigma^2 R < \text{const.} \tag{4.4}$$

It is obvious that if the theoretical value of σ turns out to be lower than σ_0 and if the possibility of brittle fracture must be eliminated, it will be necessary to replace the large vessel by several small vessels. For example, a doubling of σ calls for a fourfold reduction in R on the small vessels. To obtain the same overall capacity and maintain the same discharge as in the original spherical vessel, it will be necessary to construct 16 geometrically similar small vessels. Such a solution is hardly justifiable from an economic standpoint. However, it is much easier to avoid ERSE if the original vessel is cylindrical. In this case, with the vessel volume remaining unchanged, it will be sufficient to reduce R and increase the height of the structure. An alternative would be to bind together parallel-connected smaller-diameter pipes of the same relative thickness as the original vessel. Such a solution might prove quite workable.

D. In the example with the cable, a restriction was applied only to the cross section of its constituent elements. The number of elements could have been as large or small as necessary, depending on the specified load F. This conclusion is of fundamental importance, since it opens up the possibility of designing objects of any dimensions that would be free of ERSE. More accurately stated, it makes it possible to design structures of different sizes using load-bearing elements having a constant characteristic dimension 2r (where r is the radius of an element or strand).

Taking the above approach to the design of structures also solves another important problem — reducing the weight of the structure as a whole. The solution lies in the use of brittle lightweight materials such as glass. For example, with a density approximately one-third that of steel, a fiber of ordinary glass of grade VM-1 with a diameter 2r = 10 μm fractures at σ = 4.2 GPa. This is still within the elastic region. Here, elastic strain reaches ε = 4.8%. Composites consisting of oriented glass fibers (as the load-bearing element) and epoxy resin (as the binder) are currently in common use in various applications.

The first experimental studies of the dynamic failure of GS shells made of glass-fiber-reinforced plastics revealed a remarkable property of these composites — the absence of ERSE [34]. The wave resistances of the glass and epoxy resin used in composites of this type differ by a factor of seven. Thus, it can be expected that structures made of other oriented composites (besides glass-epoxy) will also be insensitive to ERSE if the wave resistances of their components are also sufficiently different. Subsequent systematic studies of structures of this material subjected to intensive dynamic loading confirmed the absence of ERSE and revealed that these materials have several remarkable properties under dynamic loading conditions [35, 36].

Figures 6 and 7 show the results of studies of the load-carrying capacity of GS shells of steel and glass-plastic filled with air and water, respectively [40]. The figures show the dependence of load-carrying capacity on scale (relative radius) when the shells were subjected to internal blast loading (points 1 are for a shell of steel 22K, point 2 for a glass-plastic shell). No energy-related scale effects were present when the shells failed at ξ = const, while such effects were present when ξ was increased and R was decreased. The data in the figures also illustrates the decisive role of ERSE compared to other types of scale effects [37]. In fact, if a different type of scale effect — such as the statistical effect — were to be manifest to a significant extent, its contribution would be seen most clearly in the failure of shells that were free of ERSE. As follows from Figs. 6 and 7, no such results were obtained in the experiment.

5. Statistical Scale Effect and Possible Causes of Catastrophic Brittle Fractures of Large Structures. The above examination of fracture from an energy viewpoint within the framework of the integral approach and linear fracture mechanics has physically substantiated the existence of ERSE. Other conditions being equal, these effects are described by Eq. (2.3) within the region of elastic deformation — the region that is most important for engineering purposes. However, in the overwhelming majority of studies reported in the scientific and technical literature, scale effects are described only from the viewpoint of the statistical theory of strength or its variants, leading to the relation

$$\sigma \sim V^{-1/3m} \sim L^{-1/m}, \tag{5.1}$$

where V is the volume of material in the object being considered; L is the characteristic dimension for GS objects; m is a material constant characterizing their defectiveness. As $m \to \infty$, the material approaches perfection and its strength approaches the theoretical value.

Why have energy-related scale effects been ignored to such an extent? There are several reasons: 1) the sources of explanation for scale effects are closely allied with traditional fracture criteria and the statistical theory of strength; 2) formula (5.1) is more general in form than (2.3) and, when $m = 2$, it formally includes (2.3) as a special case; 3) the value of m is found experimentally, so it is thought to more objectively reflect reality. The logical conclusion here is that the nature of scale effects is a topic suitable only for academic debate, having no relevance to practical applications.

What are the possible consequences of ignoring ERSE? In light of the fact that the statistical theory of strength is in disagreement with the empirical findings [38], the statistical approach to describing scale effects should be regarded merely as a convenient method of describing experimental results [39]. However, in this case the value of m will depend on the conditions under which specimens of the same given material are tested. Recognition of the fact that fracture is in fact the completion of work dictates (in order to obtain accurate information in tests with GS objects) that testing machines be modified to reflect this. The fact that m depends on the test conditions is usually ignored in static tests and in studies of the statistical scale effect, which leads to overestimates of this parameter. For example, a value $m > 2$ might be obtained as a result of a change in λ accompanying a change in L [9]. In this case, any increase in the parameter above $m = 2$ can conveniently be taken as arguing against the occurrence of energy-related scale effects. The question of the nature of scale effects is far from being purely of academic interest. If in fact the phenomenon is statistical, then it is necessary to find materials with a sufficiently large value of m, i.e., materials that are close to being free of defects. With the transition to the full-scale structure, having such a material would make it possible to account for the scale effect throught the introduction of a correction factor differing slightly from unity. Reality, however, is more demanding. Energy-related scale effects are in essence a consequence of the law of conservation of energy. Thus, if the value $3m = 20\text{-}100$ (instead of $3m = 6$) is used in the strength design of a large structure, it can be stated *a priori* that its strength reserves will be overestimated — with all of the consequences that may ensue from that. Thus, it can be said with confidence that the failure of large objects (pressure vessels, bridges, tankers, offshore drilling rigs, etc.) can be attributed to a failure to account for ERSE. This has been proven by pipeline fractures occurring over distances of many kilometers [7, 40]. Furthermore, the fact that GS shells of glass-plastic have fractured without the manifestation of significant scale effects and the fact that the fracture of steel shells has been accompanied by severe scale effects (see Figs. 6 and 7) can be regarded as unambiguous evidence of the leading role of ERSE in the failure of traditional materials.

6. Safety Factors and Ways of Preventing Brittle Fracture. What kind of recommendations can be made for the design of structures using fracture mechanics (FM), and is the possible manifestation of ERSE always to be considered? As was noted above, energy-related scale effects are taken into consideration when a solution can be found by the methods of FM (such as in the case of pipelines [6, 7, 40]) or by using a loading scheme employing K-calibration [41]. In the latter case, a slit is regarded as a part of the structure. However, such cases are encountered relatively infrequently. In most instances, since it is impossible to account for all defects, it is recommended that they be "spread" over the volume of the object. The material is then regarded as being free of flaws and the theory of the strength of materials is employed [38, 41]. The possible occurrence of ERSE is not considered when this recommendation is followed (especially for large objects) and the safety factor will be overstated.

We will mention two other possibilities which might lead to overestimation of strength and, thus, to brittle fracture. One stage of the design process is the selection of a material. It is reasoned that if standard specimens of the same thickness as the actual structure do not fail brittly in tests conducted by the FM methods, then it can be expected that the structure itself will not fail in a brittle manner [42, 43]. This method of excluding brittle fractures is recommended as the most reliable, due to the absence of a criterion for brittle fracture in FM [44] and the fact that it is usually not feasible to directly test full-scale objects to the point of failure.

In tests of specimens of full-scale thickness, the value of K_{1c} is determined at the moment the crack becomes unstable. Here, the following limitation is imposed on the width L of the specimen [45]:

$$L \geqslant (2{,}5 \ldots 6{,}25) K_{Ic} / \sigma_{0,2}^2. \tag{6.1}$$

A similar formula obtained from the exact solution of the problem of the nonfracture of pipelines leads to the relation

$$L \geqslant 2K_{Ic} / \sigma^2. \tag{6.2}$$

An important difference between inequalities (6.1) and (6.2) is the use of $\sigma_{0.2}$ and σ, respectively. In accordance with (6.1), the value of K_{1c} is found at $\sigma = \sigma_{0.2}$. However, brittle fracture can also occur at $\sigma < \sigma_{0.2}$. In the latter case, K_{1c} will correspond to steady-state brittle fracture of the full-scale structure. Thus, the absence of brittle fracture in specimens will not be a guarantee that the structure itself will not fail in this way.

Another possible reason for overestimation of strength as regards brittle fracture is related to the temperature dependence of K_{1c} (or G_{1c}, λ). Thus, when the determination is made by the methods used in FM, K_{1c} increases with an increase in T. Similar relations obtained by the cleavage method have the form of functions that decrease with an increase in T. The descending path of the relation is more realistic from a physical viewpoint and is consistent with the low values of unit rupture work seen for liquids relative to solids.

In light of the foregoing, it could be that, as T is increased, an autonomous steady-state fracture regime might be attained on large specimens not suitable for laboratory studies, i.e. on specimens on which any manifestation of ERSE should be more apparent. The attainment of such a regime would lead to higher rates of crack growth, localization of plastic flow to the crack-tip region, and a decrease in K_{1c}. The possibility of such fracture occurring is indicated by the fact that "....strain at the notch root in excess of 10% (with some margin of error) is seen in so-called brittle fracture under laboratory conditions, while the strain seen under working conditions does not exceed 2%" [45]. Thus, the value of G_{1c} obtained in the fracture of materials in an autonomous steady-state regime and at elevated temperatures may be considerably less than would follow from the value of K_{1c} measured by the FM methods on standard specimens at values of T above the T corresponding to cold-shortness. In other words, K_{1c} may be overstated significantly.

Are there other ways of predicting and preventing brittle fracture that are free of the above shortcomings? One potential approach was examined in [15]. Let us exclude the possibility of directly testing a large object to the point of fracture or testing it to determine its load-carrying capacity. In a number of cases, shells provided with structural elements and loaded from the inside act as the main load-bearing member of a structure. It was noted above that the satisfaction of a single necessary condition is sufficient for fracture to occur in high-rate dynamic loading. In this case, if we know the stress-strain curve of the material and have loaded a geometrically similar model of smaller size to failure, we can predict the character of fracture and load-carrying capacity of the object or its load-bearing member. As an alternative, we could also determine the maximum value of the characteristic dimension R_0 of a GS object for which the fracture process would remain plastic. For example, the authors of [15] presented results of tests of geometrically similar spherical shells with a relative wall thickness of 21% that were blast-loaded to failure. The shells were made of boiler steel (St. 22K) and had solid necks. The outside radius of the shells R was 75, 15, and 5 cm. The shells were described using a bilinear stress-strain curve for the constituent material in the form (2.1), where σ_0 = 0.5 GPa, K = 8.5 GPa, and E = 210 GPa. The integral approach was used to obtain a fracture equation that links R_i and the fracture strain ε_i with the quantity R_0:

$$R_0 = R_i [1 - K/E + K\varepsilon_i/\sigma_0]^2. \tag{6.3}$$

At $R_i > R_0$, Eq. (6.3) assumes the simpler form

$$R_0 = R_i (\sigma_i/\sigma_0)^2.$$

The value found for R_0 was ~20 cm. This method of predicting R_0 can be extended to actual $\sigma(\varepsilon)$ curves in order to account for $\sigma_0(\dot{\varepsilon})$. Figure 8, taken from [15], illustrates the given method of finding R_0. Here, points 1 in the plane $\xi(R^{-1})$ correspond to fracture of the shell, while points 2 correspond to the case when it remains intact. The Roman numerals I and II denote the regions of elastic and plastic deformation.

An example of the possibly catastrophic consequences of not allowing for ERSE was examined in [46] with reference to the design of vessels having a capacity of several thousand cubic meters and built for use at pressures up to 50 MPa.

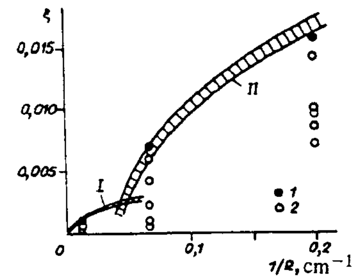

Fig. 8

7. Conclusion. In the studies discussed above, an attempt was made to describe empirical results on the dynamic fracture of GS objects from the viewpoint of the integral approach. The discussion was of a qualitative nature in a number of instances, and simplifying assumptions were used to explain the essence of the processes in question. This applies to the form of the equations chosen to describe deformation, to the discussion of the simplest stress-strain state in GS objects, and to the proposition that the physical properties — including such characteristics as σ_0 and λ — of the material of GS objects of different sizes remain constant. In the case of the two characteristics just mentioned, the assumptions made are equivalent to assuming the absence of a processing factor and supposing that the effect of $\dot{\varepsilon}$* is weak.

Let us briefly discuss the place of fracture mechanics in the integral approach. In accordance with Eq. (2.2), the quantity A is the ratio of the store of elastic energy to the work that needs to be done for a cube to fracture. In region II (see Fig. 2), $A > 1$. Brittle fracture is a possibility in this case. When $\sigma = \sigma_0$, Eq. (2.2) can be written as $L = L_0A$. Thus, an initial crack of the area L_0^2 is sufficient for the brittle fracture of a cube with an edge L placed in tension by a force σ_0L^2. However, in the integral approach the quantity L_0 is the minimum value of the characteristic dimension of the object at which brittle fracture is still possibile when $\sigma = \sigma_0$. This value is by no means small for certain materials. For example, in the uniaxial tension of a cube of copper, stainless steel (12Kh18N10T), or mild steel at $T \sim 300$ K, L_0 is measured in decimeters [2].† Thus, within the context of linear fracture mechanics, brittle fracture cannot occur in region I of the constitution diagram constructed in accordance with the integral approach (see Fig. 2) because the critical crack size must be greater than the size of the object itself. In the other limiting case $A >> 1$ — typical of materials that are brittle under normal conditions — the state of the object poses a high risk of failure. The value of A can be reduced by sharply lowering L. This also permits use of the high value of $\dot{\sigma}_0$ of brittle materials, as was demonstrated in the example of glass-fiber-reinforced plastics.

It was noted that the quantities G_{1c} and 2γ in FM are the analogs of λ in the integral approach. The question of the differences in their behavior with an increase in T was discussed in Part 6. Another important difference is the following. The values of G_{1c} and 2γ are determined in experiments in which a main crack grows, while λ is usually determined by high-rate fracture (cleavage). In the latter case, fracture is initiated and proceeds synchronously at a large number of sites. The first attempt to systematize empirical data on cleavage with GS specimens [1] showed that the exponent with σ in Eq. (4.4) has a value different than two. Later experiments confirmed this. The change in the exponent is due to the dependence of λ on the length (or time of action) of the tensile pulse [9]. Thus, if this relation is constructed in the form $\lambda \sim L^\kappa$ for geometrically similar experimental systems, then Eq. (4.4) for cleavage fracture is written in the form

$$\sigma^{2/(1-\kappa)}L = \text{const}. \tag{7.1}$$

*Thus, for steels, σ_0 changes by no more than 2-3% with a tenfold change in $\dot{\varepsilon}$. If the type of fracture being examined is not cleavage, then G_{1c} (the analog of λ) is also weakly dependent on $\dot{\varepsilon}$. According to [47], an increase of five orders in $\dot{\varepsilon}$ leads to an increase in K_{1c} by a factor of just 1.5-2. With K_{1c} depending linearly on $\dot{\varepsilon}$, an increase of one order in $\dot{\varepsilon}$ will correspond to an increase in G_{1c} by just 0.03-0.04%.

†Such a value of L_0 makes it possible, for example, to understand why a bridge in Melbourne began to fail at a crack length of approximately 3 m [3].

The quantity κ and the constant in (7.1) are characteristics of the material. They are found from tests conducted with GS equipment by looking for the plate collision velocity which leads to cleavage in the target.* For example, values of κ reported for an aluminum alloy (σ_0 = 333 MPa, σ_B = 449 MPa, E = 81 GPa) [49] and copper [9, 50] were 0.24 and 0.63. The quantities G_{1c} or 2γ should depend similarly on L. However, as was indicated in the last footnote, it is negligibly small. This fact leads us to the conclusion that it is best to use G_{1c} instead of λ in the integral approach as well when fracture occurs by the growth of a main crack.

Allowing for the above, let us compare the formulas for the brittleness threshold in the integral approach (2.3) and the length of Irwin's plastic zone in fracture mechanics [3]

$$r_y = \frac{1}{2\pi}\frac{G_{1c}E}{\sigma_0^2}. \tag{7.2}$$

The below equation is obtained from (2.3) and (7.2) to express the brittleness threshold in terms of the Irwin zone:

$$L_0 = 4\pi r_y.$$

Thus, at $\sigma = \sigma_0$, brittle fracture is possible only for a cube having the dimensions $L \geq 4\pi r_y$. It cannot occur when $L < 4\pi r_y$.

Let us summarize some of the results that have been obtained using the integral approach to describe dynamic fracture.

1. A scheme for constructing a unified theory of fracture was proposed. The states in which fracture — including brittle fracture — is impossible were indicated.

2. The decisive role of energy-related scale effects in the dynamic fracture of geometrically similar objects was demonstrated. Failure to allow for ERSE in the design and construction of the load-bearing members of large structures leads to overestimation of the safety factor. This may be one reason for the unexpected brittle fracture of such objects.

3. Certain ways of avoiding ERSE in the design of structure made of traditional materials were discussed. A series of studies of the dynamic fracture of shells made of glass-fiber-reinforced plastic made it possible to establish and theoretically substantiate a remarkable property of this material — insensitivity to ERSE.

4. The integral approach was used to describe the high-rate cleavage failure of a material initiated at multiple sites. Particular attention was given to qualitative differences in the temperature dependences of G_{1c} (or 2γ) and λ. It was suggested that the values of G_{1c} and K_{1c} obtained for high temperatures are overestimates. It was shown that λ is heavily dependent on L (or $\dot{\varepsilon}$) in the case of cleavage.

5. Features of the fracture of shells both in the elastic strain region under static loads and deep within the plastic region were examined. In the latter case, an understanding was reached of the physical nature of the dynamic peak of plastic strain exhibited by materials having a tough strength component. This same phenomenon was also described mathematically. A greater understanding was also achieved with respect to the mechanism by which shells fragment when exploded and jets formed by shaped charges break up.

6. Use of the integral approach to study pipelines makes it possible to refine the concept of service reliability to allow for dynamic loads and the properties of the pipe material, as well as to evaluate scale effects in fractures occurring under extreme abnormal loads [51].

*The value of λ for cleavage can also be determined by deliberately choosing a plate collision velocity that will lead to cleavage, regardless of the value of L. The value of σ for cleavage is determined from the recorded velocity of the free surface of the target. In this case, the value of σ will depend on L and the the chosen loading pressure [48].

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